A PRECISION TEST OF HIPPARCOS SYSTEMATICS TOWARDS THE HYADES

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ABSTRACT

We propose and apply a test that can detect any systematic errors in the Hipparcos parallaxes towards the Hyades cluster at the level of 0.3 mas. We show that the statistical parallax method subsumes the classical moving cluster methods and provides more accurate and robust estimates of the distance and the first two moments of the velocity distribution of the Hyades cluster namely, its bulk space velocity and the velocity dispersion tensor. We predict the parallaxes of Hyades cluster members using the common cluster space velocity derived from the statistical parallax method and their individual Hipparcos proper motions. We show that the parallaxes determined in this manner ($\pi_{\rm pm}$) are consistent at the 1σ level with the parallaxes ($\pi_{\rm orb}$) of three Hyades spectroscopic binary systems with orbital solutions. We find that $\langle \pi_{\rm pm} - \pi_{\rm orb} \rangle = 0.49 \pm 0.47$ mas, where the error is dominated by the errors in the orbital parallaxes. A reduction in this error would allow a test of the systematic errors in the Hipparcos parallaxes at the 0.3 mas level. We also find that the parallaxes determined using the Hipparcos proper motions and the common cluster velocity are consistent with the Hipparcos parallaxes themselves, thus confirming that the Hipparcos astrometric results towards the Hyades cluster are self consistent. Along the way, we determine the velocity of the Hyades cluster in equatorial coordinates to be $(V_x, V_y, V_z) = (-5.70 \pm 0.19, 45.67 \pm 0.11, 5.61 \pm 0.08)\ {\rm km\,s^{-1}}$ and its velocity dispersion to be $330 \pm 30\ {\rm m\,s^{-1}}$.

Subject headings: astrometry: parallaxes, methods: analytical, statistical, Galaxy: open clusters and associations: Individual(Hyades)

1. INTRODUCTION

The Hipparcos mission (ESA97) has derived accurate astrometric parameters for about 120,000 stars distributed all over the sky. The systematic errors in the parallaxes

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(proper motions) are claimed to be $\lesssim 0.1$ mas (mas yr^{-1}), while the random errors are of the order of 1 mas (mas yr^{-1}). However, recent comparisons of the distances to open clusters derived from Hipparcos parallaxes and main sequence fitting techniques show surprisingly large differences for some clusters that can be reconciled only if the systematic error in the Hipparcos parallaxes is at the level of 1 mas, at least in certain parts of the sky (Pinsonneault et al. 1998, hereafter PSSKH98, Mermilliod et al. 1997; Robichon et al. 1997). It is therefore prudent to compare the Hipparcos parallaxes with accurate parallaxes determined in an independent manner. In this paper, we propose and apply one such method which could test for systematic errors in Hipparcos astrometry towards the Hyades cluster at the level of 0.28 mas.

The statistical power of our test derives from the fact that the fractional error in the Hipparcos proper motions of the Hyades cluster members ($\sigma_\mu / \langle\mu_{\rm Hya}\rangle = 1.4\%$) is about four times smaller than the fractional error in their Hipparcos parallaxes ($\sigma_\pi / \langle\pi_{\rm Hya}\rangle = 6\%$), where we have assumed that the mean parallax and the proper motion of the Hyades cluster are $\langle\pi_{\rm Hya}\rangle = 21.5$ mas and $\langle\mu_{\rm Hya}\rangle = 111$ mas yr^{-1} respectively and their errors are $\sigma_\pi = 1.3$ mas and $\sigma_\mu = 1.5$ mas yr^{-1}. Hence, if one can accurately determine the space velocity of the Hyades cluster, one can use the Hipparcos proper motions to predict the parallaxes of the individual Hyades members more accurately than the Hipparcos parallaxes, assuming that all the members partake in this common cluster motion (to within the velocity dispersion of the cluster). The accurate parallaxes predicted in this manner can then be compared with the Hipparcos parallaxes and with parallaxes determined in an independent manner to determine the level of the systematic errors in the Hipparcos astrometry.

The basic steps of our test are as follows:

(1): We select a set of Hyades cluster members all of which are consistent with having the same velocity as the bulk motion of the cluster itself.

(2): We derive the space velocity of the Hyades cluster by combining the radial velocities of the individual cluster members, their Hipparcos proper motions and their photometric distance modulus derived from an isochrone of the Hyades main sequence. All these independent data are elegantly combined in the statistical parallax method to derive a maximum likelihood estimate of the distance scale and the first two moments of the velocity distribution of the Hyades – the bulk velocity and the velocity dispersion tensor (Hawley et al. 1986; Strugnell, Reid & Murray 1986; Popowski & Gould 1998). This procedure generalizes the classical moving cluster methods. We combine this solution with Hipparcos trigonometric parallaxes and derive a robust estimate of the bulk velocity of the cluster.

(3): We adopt this common cluster velocity to predict the parallaxes of three Hyades spectroscopic binary systems whose orbital solutions are known accurately from previous work.

(4): We compare the parallaxes predicted in this manner with the parallaxes from the orbital solutions of the three binaries to check if there are any systematic errors in the Hipparcos astrometry towards the Hyades cluster.

The outline of this paper is as follows. We explain the connection between the classical moving cluster methods and the statistical parallax method in §2. We describe our selection of Hyades cluster members and our estimate of its common space velocity in §3. In §4, we outline the procedure for estimating the parallaxes to individual cluster members using the common cluster motion and proper motions from the Hipparcos catalog. We compare the parallaxes predicted by this method with the parallaxes from the orbital solutions of the 3 binary systems in §5. We conclude in §6 by describing the future potential of this method.

2. STATISTICAL PARALLAX AS A GENERALIZED MOVING CLUSTER METHOD

In this section, we describe how the statistical parallax method is a generalized form of the classical moving cluster methods and provides a more accurate and precise distance to the Hyades cluster. First, we present the various equations describing the geometry of the cluster motion and show how the statistical parallax method subsumes the moving cluster methods. We then use some simple assumptions about the Hyades cluster to present a quantitative estimate of how much extra information about the distance is present in the statistical parallax formalism compared to the moving cluster methods.

Consider a cluster at a distance d whose bulk velocity is $\mathbf{V}$. If the radial velocity at an appropriately defined cluster center is V_r and the transverse velocity of the cluster in the plane of the sky is $\mathbf{V}_T$, we have,

$$\boldsymbol{\mu} = \frac{\mathbf{V}_T}{d}, \tag{1}$$

$$\mathbf{V}_T = \mathbf{V} - V_r\hat{\mathbf{r}}, \tag{2}$$

where $\boldsymbol{\mu}$ is the proper motion vector of the cluster center in the plane of the sky. The difference between the transverse-velocity and the proper-motion vectors ($\delta\mathbf{V}_T$ and $\delta\boldsymbol{\mu}$) of the cluster center and those of the individual cluster members are then given by

$$\delta\mathbf{V}_T = -V_r\boldsymbol{\theta}, \tag{3}$$

$$\delta\boldsymbol{\mu} = \left(\frac{\delta \mathbf{V}_T}{d}\right) - \left(\frac{\delta d}{d}\right)\boldsymbol{\mu} = -\left(\frac{V_r}{d}\right)\boldsymbol{\theta} - \left(\frac{\delta d}{d}\right)\boldsymbol{\mu}, \tag{4}$$

where $\boldsymbol{\theta}$ is the angular separation vector between the cluster center and the cluster member star in the plane of the sky, and we have assumed that $|\boldsymbol{\theta}| \ll 1$ (the small angle approximation), and $(\delta d/d) \ll 1$. This vector can be split into two components $\mu_{||}$ and $\mu_{\perp}$ along the directions that are parallel and perpendicular respectively, to the proper motion vector ($\boldsymbol{\mu}$) of the cluster in the plane of the sky. Equation (4) can then be written in terms of these components as

$$\delta\mu_{\perp} = -\left(\frac{V_r}{d}\right)\theta_{\perp}, \tag{5}$$

$$\delta\mu_{||} = -\left(\frac{V_r}{d}\right)\theta_{||} - \left(\frac{\delta d}{d}\right)\mu_{||}, \tag{6}$$

since $\mu_{\perp} = 0$, by definition. Further, we also have

$$\delta V_r = (\boldsymbol{\theta}\cdot\boldsymbol{\mu})d = \theta_{||}\mu_{||}d = \theta_{||}V_T, \tag{7}$$

where δV_r is the difference between the radial velocities of the cluster member star and the cluster center, and $V_T = |\mathbf{V}_T|$.

It is clear from the above equations that there are three independent measures of the distance to the cluster from equations (5), (6), and (7). In the classical moving cluster method, the proper motions of the individual cluster members are used to derive a convergent point on the sky. This information is combined with an average radial velocity of the cluster center to derive its distance using equation (5) (Boss 1908; Schwan 1991). Alternatively, if there are reliable radial velocities of the cluster members, equation (7) can be used to estimate the cluster distance by making use of the average proper motion of the cluster center (Detweiler et al. 1984; Gunn et al. 1988, hereafter G88). Equation (6) has not been used so far to measure the distance as it includes a combination of two terms which are degenerate in the absence of an independent estimate of δd.

All three independent estimates of the cluster distance are naturally combined in the statistical parallax method. The resultant distance is then the weighted average of the individual distances from the three equations. Since these distance estimates are independent of each other, their variances add harmonically. The weights from each of these estimates is proportional to $(d_i/\sigma_i)^2$ where d_i and $\sigma_i, (i = 1, 2, 3)$ are the distances and the errors in the distances from each of the three equations. These weights are approximately given by

$$W_1 = \left\langle \frac{(\theta_{\perp}V_r)^2}{A} \right\rangle, \tag{8}$$

$$W_2 = \left\langle \frac{(\theta_{\|} V_T)^2}{B} \right\rangle , \tag{9}$$

$$W_3 = \left\langle \frac{(\theta_{\|} V_r)^2}{C} \right\rangle , \tag{10}$$

$$W_4 = \left\langle \frac{(\theta_d V_T)^2}{C} \right\rangle , \tag{11}$$

where $A = [(d\sigma_\mu)^2 + \sigma^2]$, $B = [\sigma_r^2 + \sigma^2]$, and $C = [A + (\sigma_d \mu)^2]$. Here, σ_r and σ_μ are the errors in the radial velocities and the proper motion respectively, σ_d is the error in the distance to individual cluster members, $\theta_d = \delta d/d$, and σ is the velocity dispersion of the cluster. The weights W_1 and W_2 are the weights corresponding to the classical moving cluster methods using proper motions [eq. (5)] and radial velocities [eq. (7)] respectively, while W_3 and W_4 are the weights from the first and the second terms of equation (6).

For the purpose of illustration, we assume that for the Hyades cluster $d\sigma_\mu = 0.3$ km s^{-1}, $\sigma_r = 0.2$ km s^{-1}, $\sigma_d \mu = 0.6$ km s^{-1}, $\left\langle \theta_{\|}^2 \right\rangle = \langle \theta_\perp^2 \rangle = \langle \theta_d^2 \rangle \equiv \langle \theta^2 \rangle$, and $V_r = (5/3)V_T = 40$ km s^{-1}. This leads to $W_1 : W_2 : W_3 : W_4 = 1 : 0.50 : 0.33 : 0.12$, showing that there is significant information about the distance in the two terms of equation (6). Hence, the distance estimate using the statistical parallax method is more robust and accurate compared to that derived using the classical moving cluster methods alone. We also note that in the absence of observational errors, the fractional accuracy in the cluster distance from the statistical parallax method using N cluster member stars is given by

$$\frac{\Delta d}{d} = \left(2N\kappa^2 \left\langle \theta^2 \right\rangle \right)^{-\frac{1}{2}} , \tag{12}$$

where $\kappa = (V/\sigma)$ is the "Mach number" – the ratio of the bulk velocity of the cluster to its velocity dispersion.

3. MEMBERSHIP AND COMMON CLUSTER MOTION

We now find the bulk velocity of the Hyades cluster using the Hipparcos proper motions of the cluster members. However, a non-trivial problem here is the identification of the stars belonging to the cluster itself. We present our criteria for selecting the Hyades cluster members in §3.1 and describe our method of deriving its space velocity in §3.2.

3.1. *Cluster Membership*

We select a preliminary set of 75 Hyades cluster candidates from the list of Hyades candidates in Table 2 of Perryman et al. (1998, hereafter P98) Our selection criteria for choosing these candidates are as follows:

(1): All the candidate stars should have reliable radial velocity measurements. We correct the raw radial velocity measurements of those stars measured by Griffin et al. (1988) (stars with an entry 1 in column r of Table 2 of P98) using the procedure described by equation (12) of G88, but accounting for a sign error (P98; R.Griffin 1998, private communication).

(2): We reject all the stars that are either known or suspected to be binaries from earlier work. These are the stars with any alphabetical entry in at least one of the columns s, t or u of Table 2 of P98.

(3): We select only those stars with reliable ground based photometric measurements of both V_J and $(B-V)_J$. We use the mean values of these quantities for the candidate stars from the GCPD photometric database of Mermilliod, Mermilliod & Hauck (1997), or from the Hipparcos catalog itself if the former are not available.

(4): The candidate stars should have colors in the range $0.4 < (B-V)_J < 1.0$.

(5): We also reject any star that is flagged as a variable in the Hipparcos catalog.

We derive the photometric distance modulus $(m-M)$ to each of the 75 candidate stars in the color range $0.4 < (B-V)_J < 1.0$ by finding the difference between the apparent magnitude of the star and the absolute magnitude for its color predicted by the isochrones of the Hyades main sequence. For the adopted range in $(B-V)_J$ color, the isochrones are reliable indicators of the distance modulus up to a possible global offset. Since the Hyades isochrones have not previously been determined to high precision, we apply our selection criteria using two distinct isochrones which, as we show below, span the range of the true isochrone. First, we use the isochrones adopted by PSSKH98 and we refer the reader to that paper for further details about the construction of the isochrones. We assume a metallicity of [Fe/H] = +0.14 and an age of 600 Myr for the Hyades (P98). We use the Yale color calibration (Green 1988) to transform the isochrones from the luminosity-temperature plane to the color-magnitude plane. Second, we use the color calibration proposed by Alonso, Arribas, & Martinez-Roger (1996) which predicts a different shape for the isochrone. We find that the cluster membership is identical for both of these isochrones. We note that the two isochrones have different zero points and color dependence with the result that if the isochrones are forced to coincide at $(B-V)_J = 0.4$, they differ by about 0.3 mag at $(B-V)_J = 1.0$. We assume that the true isochrone is in the general range of these two fiducial isochrones and parametrize it by the function

$$M_V(B-V) = M_{V,\mathrm{Yale}}(B-V) + \Delta(m-M) + \alpha[(B-V)_J - 0.7] \tag{13}$$

where $\Delta(m-M)$ and α are parameters to be determined. These allow for both an offset in the zero point and a different slope for the color-magnitude relation.

For each pair of values of $\Delta(m-M)$ and α, we determine the space velocities ($\mathbf{V}_i$) of all these candidates using their photometric distance modulus, their radial velocities and their proper motions from the Hipparcos catalog. We derive a best-fit mean velocity ($\bar{\mathbf{V}}$) of their centroid and reject the stars that are gross outliers from this mean cluster motion. We compute the quantity χ^2 defined as,

$$\chi^2 = \sum_{i=1}^{N} \left(\mathbf{V}_i - \bar{\mathbf{V}}\right)^T \mathbf{C}_i^{-1} \left(\mathbf{V}_i - \bar{\mathbf{V}}\right), \tag{14}$$

where the summation is over all the N stars that remain at each iteration. The covariance matrix $\mathbf{C}_i$ for star i includes contributions from the error in the photometric distance modulus, from the error in the proper motion, and from the velocity dispersion of the cluster. The cluster velocity dispersion is between 0.2 to 0.4 $\mathrm{km\,s^{-1}}$ (G88; P98; Dravins et al. 1997) for plausible values of the cluster mass of about $300 M_\odot$ to $450 M_\odot$ and half-mass radius of the cluster of about 4 to 5 pc (Pels, Oort & Pels-Kluyver 1975; G88, P98). For the purpose of our membership selection, we assume a value of 0.4 $\mathrm{km\,s^{-1}}$. We estimate the error in the distance modulus as arising solely due to the error in the $(B-V)_J$ color and assume an average slope of 6 for the isochrone to translate this to an error in $(m-M)$. We iterate this procedure until there are no strong outliers and the velocities of all the remaining stars are consistent with a common cluster motion. The total χ^2 at the end of the iteration is 112 for 40 stars (corresponding to 117 degrees of freedom) and we reject as outlier any star whose individual contribution to χ^2 is greater than 10. All the 40 members selected from the 75 candidates lie in a tight cluster around the mean cluster motion in velocity space. Our membership selection procedure is robust to any changes in the absolute calibration of the isochrones since the relative distances between the cluster candidates are unaffected by this. However, it is sensitive to the shape of the isochrones, although in practice, we find that the cluster membership is the same for the two fiducial isochrones, despite the fact that they span a much larger range than is allowed by our fits for $\Delta(m-M)$ and α (see §3.2).

3.2. *Space velocity of the cluster*

We determine the common space velocity of the cluster from the velocities of all the 40 cluster members selected by the procedure described in §3.1. We evaluate the χ^2 [as defined in eq. (14)] in a dense grid of points in the space of the five parameters namely, $\Delta(m-M)$, α and the three components of the bulk velocity (in equatorial coordinates) of the cluster ($\bar{\mathbf{V}}$). We fit this to a quadratic in the 5 parameters to find the best fit values and the covariance matrix of the parameters at the minimum of the χ^2 surface. These best fit values are $\alpha = 0.26 \pm 0.05, \Delta(m-M) = 0.12 \pm 0.04$ mag, $V_x = -5.66 \pm 0.36$ km s^{-1}, $V_y = -45.65 \pm 0.21$ km s^{-1}, and $V_z = -5.61 \pm 0.12$ km s^{-1}. We adopt this as our initial guess of the cluster velocity to predict the cluster space velocity using the statistical parallax method. We estimate the components of the cluster space velocity in a coordinate system that is oriented such that the x-axis is along the radial direction of the center of mass (V_r), the y-axis is along the direction perpendicular to the proper motion of the cluster in the plane of the sky ($V_\perp$) and the z-axis is parallel to the proper motion of the cluster in the plane of the sky ($V_{||}$). By definition, V_r is the radial velocity of the cluster, $V_{||}$ is its velocity in the plane of the sky, and $V_\perp$ is zero.

To compute the the photometric distance to each star, we fix the slope-correction at its best-fit value of $\alpha = 0.26$, and we adopt the best-fit zero point $\Delta(m-M) = 0.12$ mag for our fiducial distance scale. In the modern version of the statistical parallax method as described by Popowski & Gould (1998), one uses the maximum likelihood procedure to simultaneously solve for ten different parameters viz., the distance scaling factor relative to a fiducial distance scale (η), the three components of the bulk velocity of the cluster ($V_r, V_\perp$ and $V_{||}$) and the six independent components of the second moments of its velocity distribution – the three diagonal terms corresponding to the square of the velocity dispersion in the three directions ($\sigma_r^2, \sigma_\perp^2$ and $\sigma_{||}^2$) and the three unique off-diagonal terms ($\sigma_{r\perp}^2, \sigma_{r||}^2$ and $\sigma_{\perp||}^2$). We assume an isotropic velocity dispersion tensor of the Hyades with the result that the three independent off-diagonal terms are constrained to be zero. For an assumed level of errors in the radial velocities, the proper motions, and the distance to individual stars, the statistical parallax method derives a maximum likelihood estimate of the cluster velocity dispersions in the three mutually perpendicular directions. However, the errors in the distance to each star affect only the parallel dispersion $\sigma_{||}$ while the estimates of the velocity dispersions in the radial and the perpendicular directions (σ_r and $\sigma_\perp$ respectively) are independent of the distance errors. Therefore, we begin by constraining the velocity dispersion in these two directions to have the same value. The velocity dispersion in the parallel direction ($\sigma_{||}$) now includes contributions from both the intrinsic velocity dispersion of the cluster and the dispersion arising from a possibly wrong estimate of the distance errors. We find that we need to add an error of 0.049 mag in quadrature to our original errors (as listed in the photometric sources) in the photometric distance modulus so that the velocity dispersion in the parallel direction ($\sigma_{||}$) becomes equal to the velocity dispersions in the other two directions. This velocity dispersion is equal to 330 ± 30 m s^{-1} and we adopt this value throughout our analysis.

The statistical parallax method finds the maximum likelihood solution of the five independent parameters namely, $\mathbf{p} = (\eta, V_r, V_\perp, V_{||}, \sigma_r^2)$ subject to the constraint of an isotropic velocity dispersion tensor. This solution, using only the photometric distances to individual Hyades cluster stars is given by $\mathbf{p}(\text{phot}) = [1.0172 \pm 0.0181, 39.33 \pm 0.06\ \text{km s}^{-1}, 0.00 \pm 0.08\ \text{km s}^{-1}, 24.90 \pm 0.45\ \text{km s}^{-1}, 0.1075 \pm 0.0271\ (\text{km s}^{-1})^2]$. An accurate estimate of the velocity $V_{||}$ is crucial in determining the parallaxes of Hyades members from their individual proper motions alone (see §4). The error in $V_{||}$ is dominated by the error in the distance scale η, and this can be reduced if we include independent distance information to individual Hyades stars from their Hipparcos parallaxes. However, the two distance estimates (photometric distance moduli normalized by the statistical parallax solution and the Hipparcos parallaxes) can be combined to yield more accurate distances to individual Hyades members only if there are no systematic differences between them.

To check if the normalized photometric distance moduli and the Hipparcos parallaxes are consistent with each other, we compute for each cluster member, the quantity

$$\eta_i = \frac{\pi_{\text{phot},i}}{\pi_{\text{Hip},i}} \left(1 + x_i^2\right), \tag{15}$$

where

$$x_i = \frac{\sigma_{\pi,\text{Hip},i}}{\pi_{\text{phot},i}}. \tag{16}$$

Since the errors of the Hipparcos parallaxes are uniformly distributed, the quadratic correction term x_i^2 is required to ensure that the two sides of equation (15) have the same mean value (Lutz & Kelker 1973; Smith & Eichhorn 1996). The error in η_i is given by

$$\sigma_{\eta,i} = \left(\frac{\sigma_{\pi,\text{Hip}}}{\pi_{\text{phot}}}\right)_i \eta_i. \tag{17}$$

We find that the mean value of η for all the cluster members is given by $\eta_{\text{Hip}} = 0.9995 \pm 0.0083$. This is consistent with the value of $\eta_{\text{phot}} = 1.0172 \pm 0.0181$ derived using the Hipparcos proper motions alone, showing that:

(1): The distances derived using the Hipparcos proper motions are consistent with the Hipparcos parallaxes. This is a semi-independent test of the Hipparcos parallaxes and proves that the Hipparcos parallaxes and proper motions are self consistent.

(2): The distances derived independently from the Hipparcos proper motions and the Hipparcos parallaxes can be combined in a consistent manner to derive a more accurate distance to the individual Hyades members.

Since the Hipparcos parallaxes are consistent with the normalized photometric distances to individual stars, we can, in principle, combine the two independent distance estimates by adding them together weighted by the inverses of their covariance matrices. In practice, we include the Hipparcos parallax distances to individual stars in the statistical parallax algorithm itself and derive a more accurate solution for the five independent parameters. This solution, combining both the Hipparcos proper motions and the Hipparcos parallaxes is given by $\mathbf{p}(\mathrm{phot}+\mathrm{Hip}) = [1.0026 \pm 0.0075, 39.33 \pm 0.06\ \mathrm{km\,s^{-1}}, 0.00 \pm 0.08\ \mathrm{km\,s^{-1}}, 24.55 \pm 0.21\ \mathrm{km\,s^{-1}}, 0.1102 \pm 0.0272\ (\mathrm{km\,s^{-1}})^2]$ and the correlation coefficient matrix is

$$\begin{pmatrix} 1.0000 & 0.0229 & -0.0357 & 0.8350 & -0.0678 \\ 0.0229 & 1.0000 & 0.0087 & 0.0191 & 0.0057 \\ -0.0357 & 0.0087 & 1.0000 & -0.2115 & 0.0302 \\ 0.8350 & 0.0191 & -0.2115 & 1.0000 & -0.0721 \\ -0.0678 & 0.0057 & 0.0302 & -0.0721 & 1.0000 \end{pmatrix}. \tag{18}$$

It is immediately obvious that including the Hipparcos parallaxes as independent data on the distance to the Hyades members significantly reduces the error in $V_{\|}$. The fact that the maximum likelihood value of η is greater than one means that the photometric distance (our fiducial value) is an underestimate of the true distances by a factor $(\eta - 1) = 0.26\%$. The space velocity of the cluster in equatorial coordinates is $(V_x, V_y, V_z) = (-5.70 \pm 0.19, 45.67 \pm 0.11, 5.61 \pm 0.08)\ \ \mathrm{km\,s^{-1}}$ and the matrix of correlation coefficients is

$$\begin{pmatrix} 1.0000 & -0.7730 & 0.3851 \\ -0.7730 & 1.0000 & -0.4005 \\ 0.3851 & -0.4005 & 1.0000 \end{pmatrix}. \tag{19}$$

This is our best estimate of the bulk velocity of the cluster and we shall use it in the remainder of the paper to predict the parallaxes from the Hipparcos proper motions of individual stars.

We show the velocities of the Hyades cluster candidates in Figure 1. The first three panels (a)-(c) show the velocities computed using the photometric distance modulus to each star and the Hipparcos proper motions, while panel (d) shows the velocities computed using the Hipparcos parallaxes and proper motions. The crosses show the velocity components of the 40 cluster members while the open circles show those of the non-members. The smaller scatter in velocities of the members in panel (c) compared to that in panel (d) shows that the photometric distance moduli lead to a much tighter core in velocity space and hence a cleaner separation between the members and the non-members, compared to using distances inferred from Hipparcos parallaxes. The solid circle in all the panels shows the bulk velocity of the cluster. For this space velocity of the cluster, the total χ^2 is 143 for 40 stars (corresponding to 115 degrees of freedom) demonstrating that our estimates of the errors for the various quantities and of the cluster velocity dispersion are reasonable. The centroid is at a distance of $|\bar{\mathbf{r}}| = 46.89 \pm 0.35$ pc and its equatorial coordinates are $\alpha = 04^h 24^m 42^s, \delta = 17°32'.7$ (2000). This is also the direction of the radial velocity of the cluster center, i.e, the direction of V_r. The motion of the cluster in the plane of the sky is towards the direction 105°28'.7 East of North.

4. PARALLAX FROM PROPER MOTION

We adopt the cluster space velocity derived above to predict the parallaxes of the 40 member stars using their proper motions from the Hipparcos catalog. The parallax of any cluster member that has the same space velocity as the cluster is given by

$$\pi_{\mathrm{pm},i} = \frac{\left\langle (\mathbf{V_t})_i | \mathbf{C}_i^{-1} | \boldsymbol{\mu}_i \right\rangle}{\left\langle (\mathbf{V_t})_i | \mathbf{C}_i^{-1} | (\mathbf{V_t})_i \right\rangle} \tag{20}$$

where $(\mathbf{V_t})_i = \mathbf{V_c} - (\hat{\mathbf{r}}_i \cdot \mathbf{V_c})\hat{\mathbf{r}}_i$ is the transverse velocity of the cluster in the plane of the sky at the position of the star i, $\boldsymbol{\mu}_i$ is its proper motion from the Hipparcos catalog and $\mathbf{C}_i$, the covariance matrix, is the sum of the proper motion error tensor of star i and the velocity dispersion tensor divided by the square of the distance. We have employed Dirac notation,

$$\langle X|\mathcal{O}|Z\rangle = \sum_{i,j} X_i \mathcal{O}_{ij} Z_j. \tag{21}$$

The error in $\pi_{\mathrm{pm},i}$ is equal to $\left\langle (\mathbf{V_t})_i | \mathbf{C}_i^{-1} | (\mathbf{V_t})_i \right\rangle^{1/2}$.

We show the difference between the photometric parallax (π_{phot}) and the parallax determined assuming a common space velocity for all the cluster members (π_{pm}) in Figure 2. We have scaled all the photometric distances by the factor $\eta_{\mathrm{phot+Hip}} = 1.0026$ so that the mean value of this difference should be equal to zero. The horizontal error bars show the error in photometric parallax, while the vertical error bars show the uncertainty in π_{pm} alone. We find that the mean offset between the two parallaxes is given by

$$\langle \pi_{\mathrm{phot}} - \pi_{\mathrm{pm}} \rangle = -0.009 \pm 0.108 \text{ mas}, \tag{22}$$

thus confirming that there are no internal inconsistencies in our method of predicting the parallaxes from the proper motions.

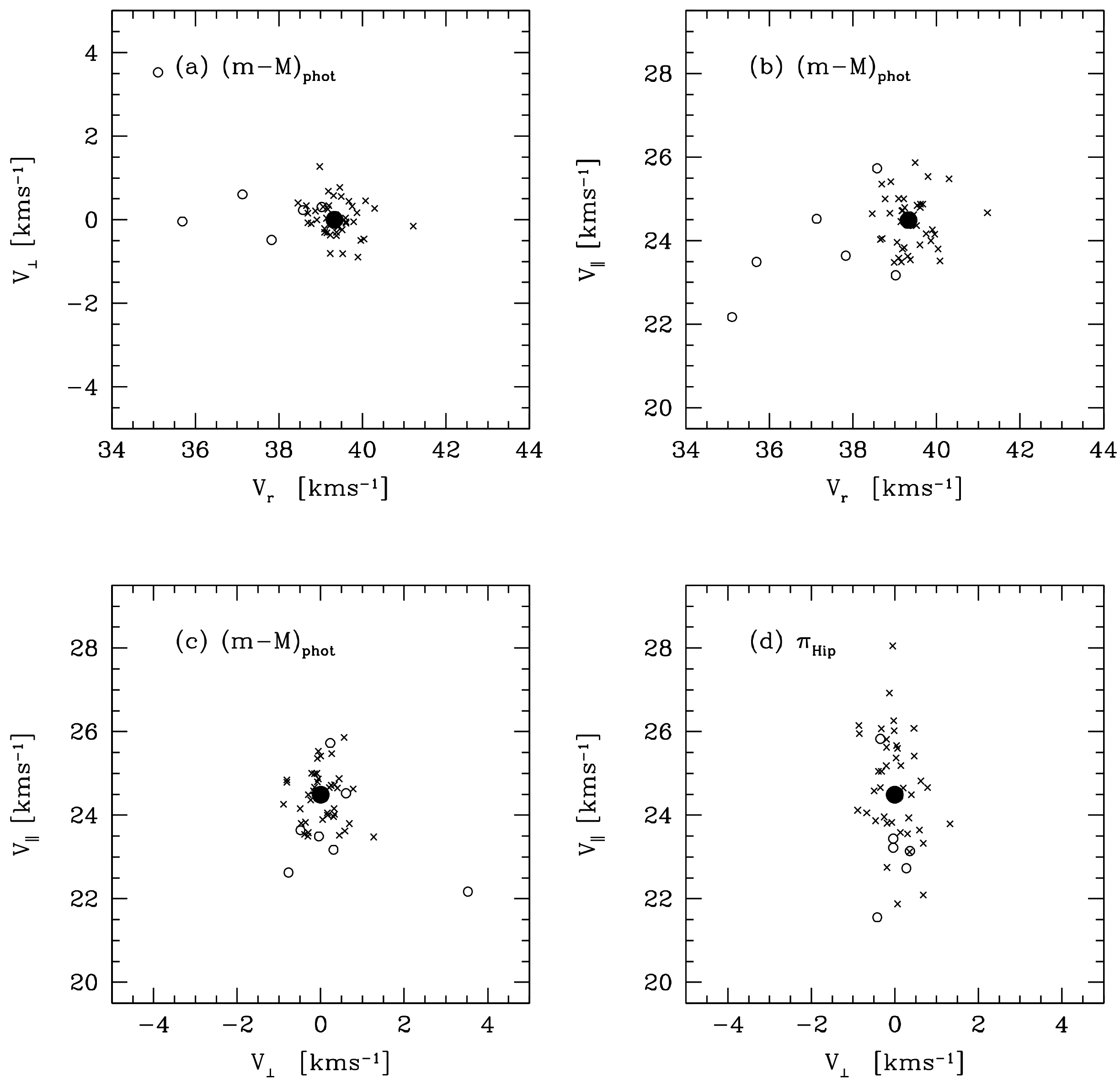

Fig. 1.— Velocities of the Hyades cluster candidates. The velocities in panels (a)-(c) are computed using the photometric distance to each star, while those in panel (d) are computed using the distance inferred from the Hipparcos parallax. (a) V_r and $V_\perp$ (b) V_r and $V_\parallel$ (c) $V_\perp$ and $V_\parallel$ and (d) $V_\perp$ and $V_\parallel$. The component V_r is the component of the velocity in the radial direction of the centroid, while $V_\perp$ and $V_\parallel$ represent the velocity components perpendicular and parallel to the direction of the proper motion of the cluster in the plane of the sky. The crosses are the velocity components of the 40 cluster members while the open circles are those of the non-members. The solid circle in each panel shows the mean motion of the cluster.

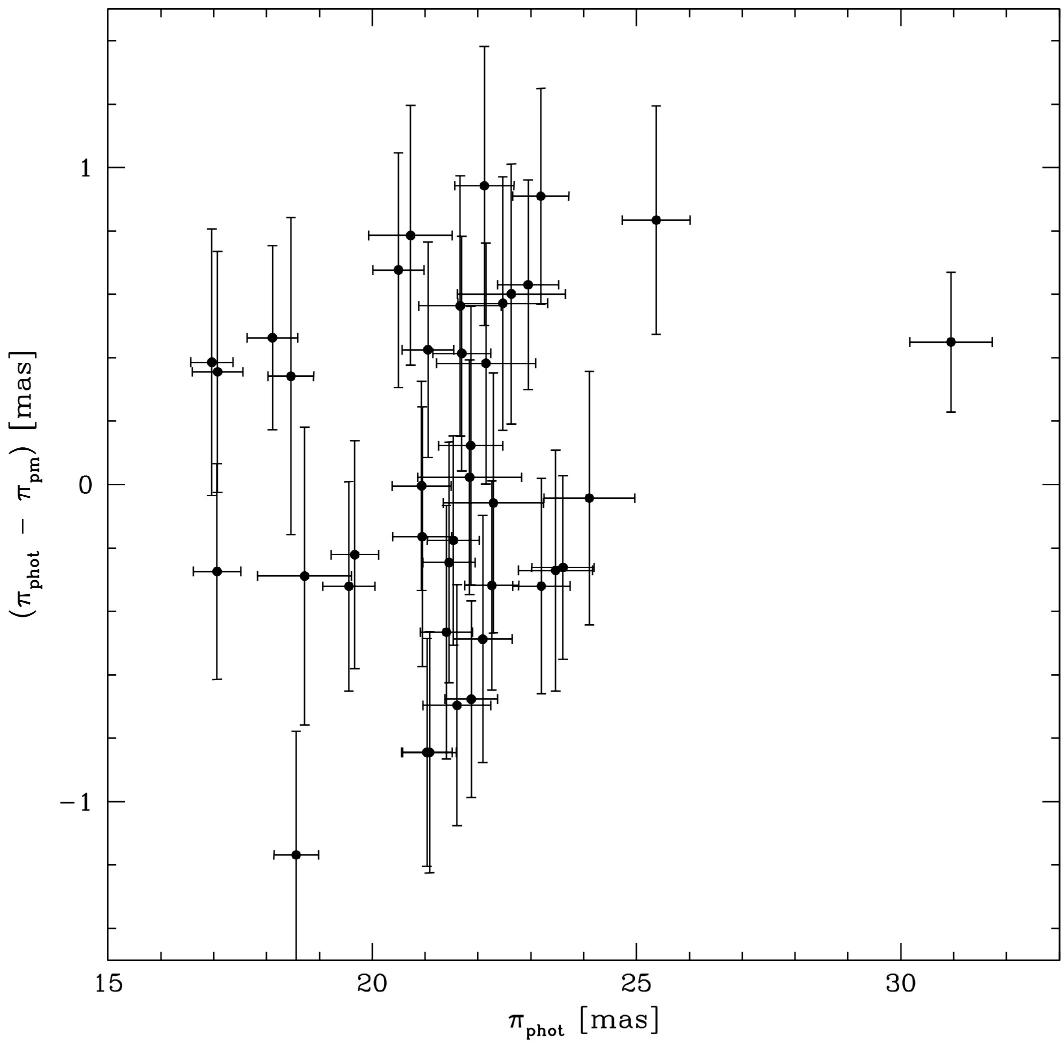

Fig. 2.— Difference between the photometric parallax ($\pi_{\rm phot}$) and the parallax predicted assuming a common space velocity for the cluster members ($\pi_{\rm pm}$). All the photometric distances have been scaled by the quantity $\eta_{\rm phot+Hip} = 1.0026$ so that the mean value of this difference should be zero. The horizontal error bars show the error in $\pi_{\rm phot}$, while the vertical error bars show the uncertainty in $\pi_{\rm pm}$ alone, i.e, the error in $(\pi_{\rm phot} - \pi_{\rm pm})$ is the quadrature sum of the two error bars.

5. PARALLAXES OF BINARY SYSTEMS

We predict the parallaxes of the 3 binary systems 51 Tauri (HIP 20087), 70 Tauri (HIP 20661) and θ^2 Tauri (HIP 20894) using the cluster space velocity ($\mathbf{V_c}$) determined above and their individual Hipparcos proper motions. The full orbital solutions of these 3 spectroscopic binary systems have been derived by Torres, Stefanik & Latham (1997a, 1997b, 1997c, hereafter T97a, T97b and T97c). The Hipparcos proper motions refer to the center of mass for HIP 20081, while they refer to the motion of the photocenter for the other two systems. HIP 20894 whose semi-major axis is less than $0\farcs1$ is listed as a variable single star in the Hipparcos catalog. We compute the proper motions of the center of mass of HIP 20661 and HIP 20894 using the spectroscopic-astrometric orbital solutions for these 2 binary systems provided by T97b and T97c respectively. In Table 1, we list the proper motions of the center of mass of all the 3 systems, their parallaxes from Hipparcos, their parallaxes from their proper motions, and their orbital parallaxes. There are two important features in the errors of the different parallaxes in Table 1.

(1): The error in the parallaxes determined from the individual proper motions of the binary systems and the common space velocity of the cluster [σ_π(pm) in column 7 of Table 1] is almost a factor of three smaller than the error in the Hipparcos parallaxes.

(2): The errors in the orbital parallaxes of these binary systems [σ_π(orb) in column 9 of Table 1] are about twice as large as the errors in the proper-motion parallaxes.

The mean difference (weighted by the inverse square errors) between the proper-motion parallaxes and the orbital parallaxes for these three binary systems is,

$$\langle \pi_{\rm pm} - \pi_{\rm orb} \rangle = 0.49 \pm 0.43 \text{ mas.} \tag{23}$$

The error in this difference is dominated by the error in the orbital parallaxes. From Table 1, we find that the error in the individual proper-motion parallaxes are each of the order

Table 1: Astrometry of the 3 spectroscopic binary systems with orbital parallaxes.

HIP ID	$\mu_\alpha \cos(\delta)$ (mas yr^{-1})	μ_δ (mas yr^{-1})	$\pi_{\rm Hip}$ (mas)	σ_π(Hip) (mas)	$\pi_{\rm pm}$ (mas)	σ_π(pm) (mas)	$\pi_{\rm orb}$ (mas)	σ_π(orb) (mas)
20087	96.42	-33.92	18.25	0.82	18.42	0.30	17.92	0.58
20661	104.97	-26.67	21.47	0.97	21.31	0.36	21.44	0.67
20894	108.80	-26.35	21.89	0.83	22.46	0.37	21.22	0.76

of 0.3 to 0.35 mas. Hence, it appears possible in principle to detect any systematic errors in the Hipparcos parallaxes at the level of $\sigma_{\rm sys} = \left(\sum 1/\sigma^2_{\rm orb}\right)^{-1/2} = 0.2$ mas if the orbital parallax errors could be reduced below this level.

However, there is another source of error in determining the proper-motion parallaxes. This arises from the error in the component of the cluster space velocity itself in the direction of its proper motion in the plane of the sky, i.e, in the component $V_{\|}$. This error is equal to $\langle \pi_{\rm Hya} \rangle \, (\sigma_{V_{\|}}/V_{\|}) = 0.19$ mas where we have again assumed that the mean parallax of the Hyades cluster is 21.5 mas. Adding this error in quadrature to the errors determined above, we find that $\langle \pi_{\rm pm} - \pi_{\rm orb} \rangle = 0.49 \pm 0.47$ mas. The irreducible error in this method $\sigma_{\rm sys}$ is now 0.28 mas, still considerably smaller than the errors in the currently available orbital parallaxes.

6. CONCLUSIONS

Our main conclusions are as follows:

(1): The distances to the Hyades members using the Hipparcos proper motions alone is consistent with the Hipparcos parallaxes, thus providing a semi-independent check of the self-consistency of Hipparcos astrometry.

(2): The bulk velocity of the Hyades cluster in equatorial coordinates is $(V_x, V_y, V_z) = (-5.70 \pm 0.19, 45.67 \pm 0.11, 5.61 \pm 0.08)$ km s^{-1} and the velocity dispersion of the cluster is 330 ± 30 m s^{-1}.

(3): The Hipparcos parallaxes of the three Hyades binary systems are consistent at the 1σ level with the parallaxes from their orbital solutions. Hence, the systematic error in the Hipparcos parallaxes towards the Hyades cluster is less than 0.47 mas.

(4): The test proposed in this paper can, in principle, detect any systematic error greater than 0.3 mas in the Hipparcos parallaxes towards the Hyades cluster. The dominant factor that currently limits a check at this level is the "large" errors in the orbital parallaxes of the binary systems.

It follows from the last two points that a more accurate estimate of the binary orbital parallaxes would enable a better determination of the systematic errors (or the lack thereof) in the Hipparcos astrometry towards the Hyades cluster.

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REFERENCES

Alonso, A., Arribas, S., & Martinez-Roger, C. 1996, A&A313, 873

Boss, L., 1908, AJ, 26, 31

Dravins, D., Lindegren, L., Madsen, S., Holmberg, J. 1997, in Hipparcos Venice '97, eds. B. Battrick & M. A. C. Perryman (ESA), 643

ESA 1997, The Hipparcos and Tycho Catalogues, ESA SP-1200

Detweiler, H.L., Yoss, K.M., Radick, R.R. & Becker, S.A., 1984, AJ, 89, 1038

Hawley, S.L., Jeffreys, W.H., Barnes, T.G.III, & Wan, L., 1986, ApJ, 302, 626

Green, E.M. 1988, in Calibration of Stellar Ages, ed. A.G.D. Phillip (Schenectady, NY:Davis), 81

Griffin, R.F., Gunn, J.E., Zimmerman, B.A. & Griffin, R.E.M., 1988, AJ, 96, 172

Gunn, J.E., Griffin, R.F., Griffin, R.E.M. & Zimmerman, B.A., 1988, AJ, 96, 198 (G88)

Lutz, T.E. & Kelker, D.H., 1973, PASP, 85, 573

Mermilliod J.-C., Mermilliod M., Hauck B. 1997, A&AS, 124, 349

Mermilliod, J.-C., Turon, C., Robichon, N., Arenou, F.,& Lebreton, Y. 1997, in Hipparcos Venice '97, eds. B. Battrick & M. A. C. Perryman (ESA), 643

Popowski, P. & Gould, A., 1998, ApJ, 506, 000

Pels, G., Oort, J.H., & Pels-Kluyver, H.A., 1975, A&A, 43, 423

Perryman, M. A. C., Brown, A.G.A., Lebreton, Y., Gomez, A., Turon, C., Cayrel de Strobel, G., Mermilliod, J.-C., Robichon, N., Kovalevsky, J., & Crifo, F. 1998, A&A, 331, 81 (P98)

Pinsonneault, M., H., Stauffer, J., Soderblom, D.R., King, J.R. & Hanson, R.B., 1998, ApJ, in press (PSSKH98)

Robichon, N., Arenou, F., Turon, C., Mermilliod, J.-C.,& Lebreton, Y. 1997, in Hipparcos Venice '97, eds. B. Battrick & M. A. C. Perryman (ESA), 567

Schwan, H., 1991, A&A, 243, 386

Smith, H. & Eichhorn, H., 1996, MNRAS, 281, 211

Strugnell, P., Reid, N., & Murray, C.A., 1986, MNRAS, 220, 413

Torres, G., Stefanik, R.P. & Latham, D.W., 1997, ApJ, 474, 256 (T97a)

Torres, G., Stefanik, R.P. & Latham, D.W., 1997, ApJ, 479, 268 (T97b)

Torres, G., Stefanik, R.P. & Latham, D.W., 1997, ApJ, 485, 167 (T97c)